

Name:

Class Period:

### First Semester Day 1 Review: Equations

#### Slope and Rate of Change

##### Definitions

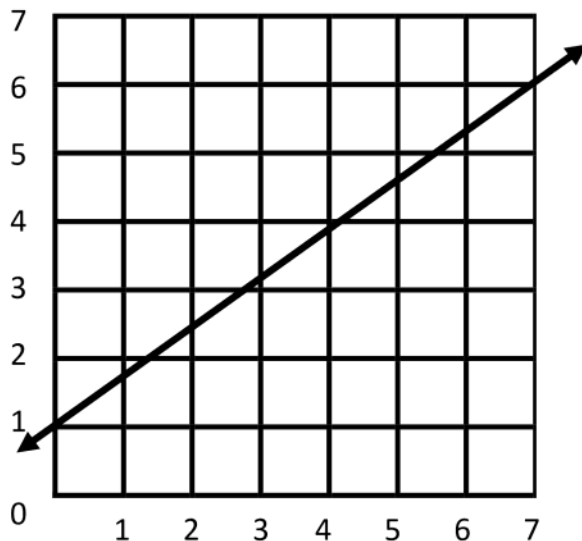
The rate of change =  $\frac{\text{change in the dependent variable}}{\text{change in the independent variable}}$

The slope =  $\frac{\text{change in } y}{\text{change in } x} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$ , where

$x_2 - x_1 \neq \text{zero}$  and the coordinates are from the points:  $(x_1, y_1)$  and  $(x_2, y_2)$ .

##### Example 1

Find the slope of the line from the graph below.



First Point:  $(0, 1)$

Second Point:  $(7, 6)$

Change in  $y$ :  $6 - 1 = 5$

Change in  $x$ :  $7 - 0 = 7$

$$\text{Slope: } \frac{\Delta y}{\Delta x} = \frac{5}{7}$$

### Example 2

Find the slope of a linear equation that passes through the points  $(-2,1)$  and  $(6,7)$ .

$$\text{Recall: } \text{slope} = \frac{y_2 - y_1}{x_2 - x_1}$$

$$\text{Replace variables with values: } = \frac{7 - 1}{6 - (-2)}$$

$$\text{Simplify: } = \frac{6}{8}$$

### **Slope-Intercept Form of a Linear Equation**

#### Definition

The slope-intercept form of a linear equation is  $y = mx + b$ , where  $m$  is the slope and  $b$  is the y-intercept.

### Example 3

Write in slope-intercept form the equation of the line with a slope of  $\frac{3}{8}$  and a y-intercept  $(0,6)$ .

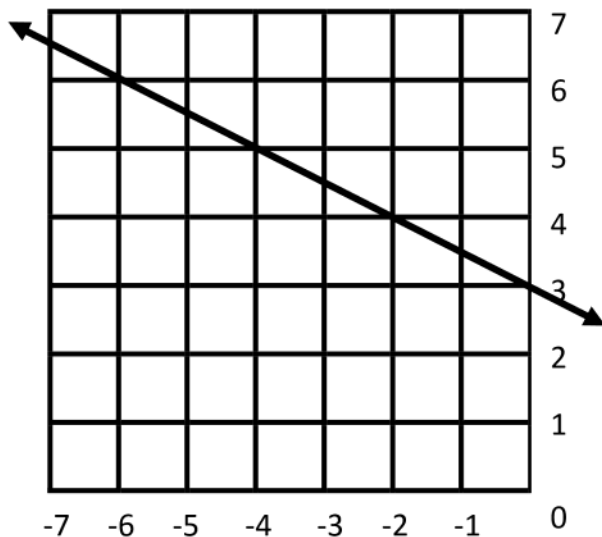
$$\text{Slope: } \frac{3}{8}$$

$$\text{y-intercept: } (0,6)$$

$$\text{Equation: } y = \frac{3}{8}x + 6$$

### Example 4

Write the equation of the line represented in the graph below in slope-intercept form.



$$(x_1, y_1): (0, 3)$$

$$(x_2, y_2): (-2, 4)$$

$$\Delta y: 4 - 3 = 1$$

$$\Delta x: -2 - 0 = -2$$

$$\text{Slope: } \frac{\Delta y}{\Delta x} = \frac{1}{-2} = -\frac{1}{2}$$

$$\text{Slope-Intercept Form: } y = -\frac{1}{2}x + 3$$

### Point-Slope Form of a Linear Equation

#### Definition

The point-slope form of a linear equation of a nonvertical line that passes through the point  $(x_1, y_1)$  and has slope  $m$  is  $(y - y_1) = m(x - x_1)$ .

### Example 5

Write in point-slope form the equation of the line with a slope of  $-3$  and passes through the point  $(-1, 7)$ .

$$m = -\frac{3}{1}$$

$$x_1 = -1$$

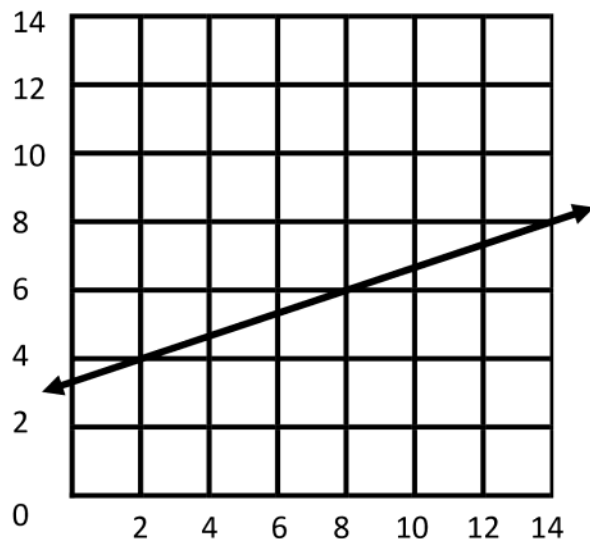
$$y_1 = 7$$

$$(y - 7) = -\frac{3}{1}(x - (-1))$$

$$(y - 7) = -\frac{3}{1}(x + 1)$$

### Example 6

Write the equation of the line represented in the graph below in point-slope form.



$$(x_1, y_1): (2, 4)$$

$$(x_2, y_2): (8, 6)$$

$$\Delta y: 6 - 4 = 2$$

$$\Delta x: 8 - 2 = 6$$

$$\text{Slope: } \frac{\Delta y}{\Delta x} = \frac{2}{6} = \frac{1}{3}$$

$$\text{Point-Slope Form: } (y - 4) = \frac{1}{3}(x - 2)$$

### Standard Form of a Linear Equation

#### Definition

The standard form of a linear equation is  $Ax + By = C$ , where  $A$ ,  $B$ , and  $C$  are real integers, and  $A$  and  $B$  are not both zero.

### Example 7

Write  $y = \frac{3}{4}x + 2$  in standard form.

$$\text{Original Equation: } y = \frac{3}{4}x + 2$$

$$\text{Move all variables to the left: } -\frac{3}{4}x + y = \left(\frac{3}{4}x - \frac{3}{4}x\right) + 2$$

$$\text{Eliminate the fraction: } \left(-\frac{3}{4}x + y = 2\right) \times 4$$

$$\text{Standard Form: } -3x + 4y = 8$$

### Example 8

Write  $(y + 3) = -\frac{1}{2}(x - 5)$  in standard form.

Original Equation:  $(y + 3) = -\frac{1}{2}(x - 5)$

Use the Distributive Property:  $y + 3 = -\frac{1}{2}x - \left(\frac{1}{2} \times (-5)\right)$

Simplify:  $y + 3 = -\frac{1}{2}x + \frac{5}{2}$

Move all variables to the left:  $\frac{1}{2}x + y + 3 = \left(-\frac{1}{2}x + \frac{1}{2}x\right) + \frac{5}{2}$

Move all constants to the right:  $\frac{1}{2}x + y + \cancel{(3 - 3)} = \frac{5}{2} + 3$

Combine like terms:  $\frac{1}{2}x + y = \frac{5}{2} + \frac{6}{2}$

Eliminate fraction:  $\left(\frac{1}{2}x + y = \frac{11}{2}\right) \times 2$

Standard Form:  $x + 2y = 11$

## Parallel Lines

### Definition

Parallel lines are lines in the same plane that never intersect.

### Property

Nonvertical lines are parallel if they have the same slope and different y-intercepts. Any two vertical lines are parallel.

### Example 9

Write an equation for the line that contains (5, 1) and is parallel to

$$y = \frac{3}{5}x - 4.$$

Write in point-slope form:

$$(y - y_1) = m(x - x_1)$$

Substitute:

$$y - 1 = \frac{3}{5}(x - 5)$$

Use the Distributive

$$y - 1 = \frac{3}{5}x - \frac{15}{5}$$

Property:

Isolate y:

$$y + \cancel{(-1)} = \frac{3}{5}x - 3 + 1$$

Slope-Intercept form:

$$y = \frac{3}{5}x - 2$$

## Perpendicular Lines

### Definitions

Perpendicular lines are lines that intersect to form 90°/right angles.

The product of two numbers is -1 if one number is the negative reciprocal of the other.

$$\text{For example: } \frac{1}{2} \times \left(-\frac{2}{1}\right) = -\frac{1 \times 2}{2 \times 1} = -\frac{2}{2} = -1$$

### Property

Two lines are perpendicular if the product of their slopes is -1.

A vertical and a horizontal lines are also perpendicular.

Example 10

Find the equation of the line that contains  $(0, -2)$  and is perpendicular to  $y = 5x + 3$ .

Find the negative reciprocal (r):

$$5 \times r = -1$$

Solve for r:

$$r = -\frac{1}{5}$$

Write in slope-intercept form:

$$y = mx + b$$

Substitute:

$$y = -\frac{1}{5}x + (-2)$$

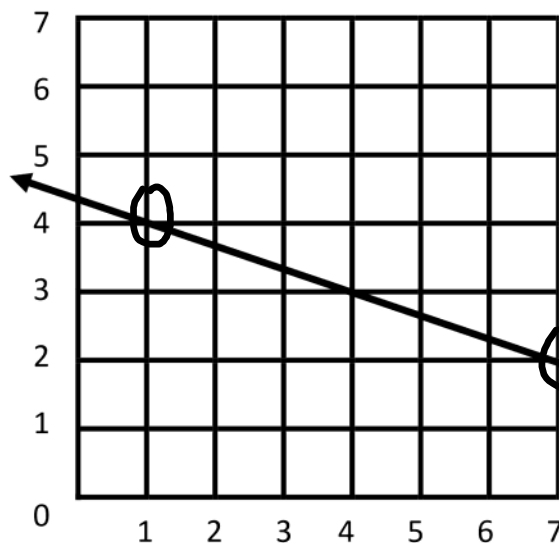
Slope-Intercept form:

$$y = -\frac{1}{5}x - 2$$

**Classwork**

Problem 1

Find the slope of the line from the graph below.



$$(x_1, y_1) = (1, 4)$$

$$(x_2, y_2) = (7, 2)$$

$$\Delta y = y_2 - y_1 = 2 - 4 = -2$$

$$\Delta x = x_2 - x_1 = 7 - 1 = 6$$

$$m = \frac{\Delta y}{\Delta x} = \frac{-2}{6} = -\frac{1}{3}$$

Problem 2

Find the slope of a linear equation that passes through the points (3, 2) and (5, 6).

$$\begin{array}{cc} \parallel & \parallel \\ (x_1, y_1) & (x_2, y_2) \end{array}$$

$$\begin{array}{l} \Delta y = y_2 - y_1 = 6 - 2 = 4 \\ \Delta x = x_2 - x_1 = 5 - 3 = 2 \end{array} \quad \left\{ m = \frac{\Delta y}{\Delta x} = \frac{4}{2} = 2 \right.$$

Problem 3

Write in slope-intercept form the equation of the line with a slope of  $\frac{2}{9}$  and a y-intercept (0, 3).

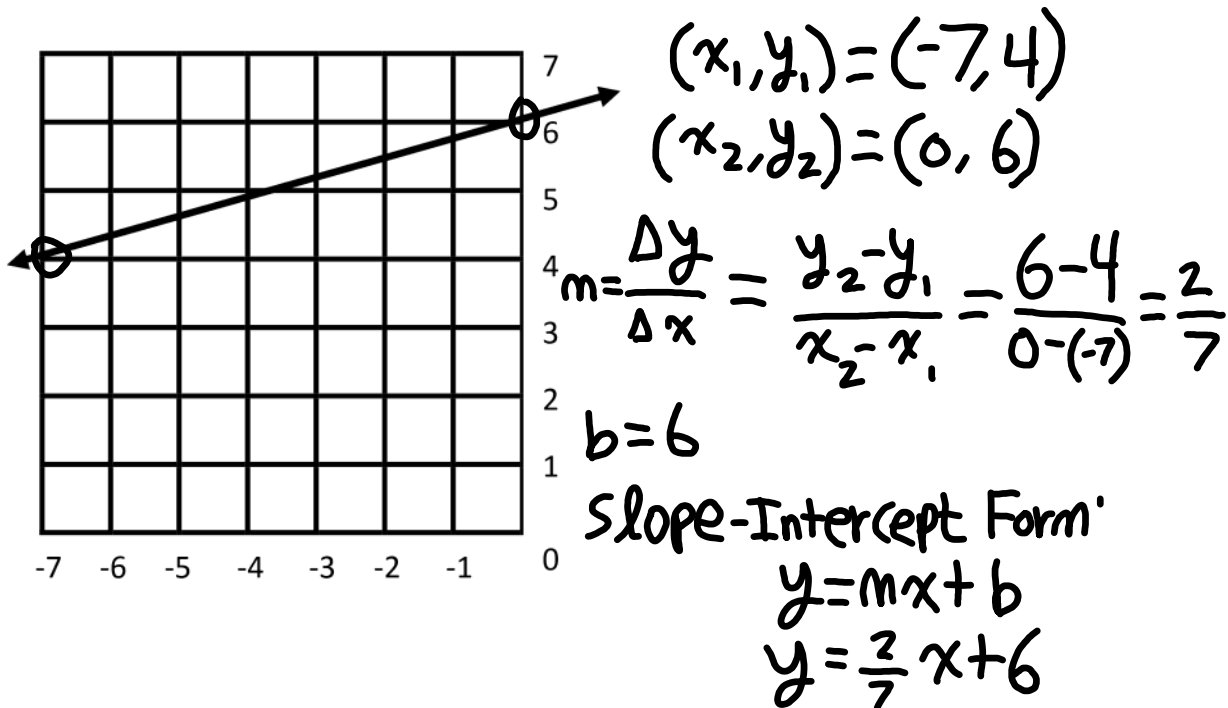
$$m = \frac{2}{9}, b = 3$$

Slope-Intercept Form  $y = mx + b$   
 $y = \frac{2}{9}x + 3$



Problem 4

Write the equation of the line represented in the graph below in slope-intercept form.



Problem 5

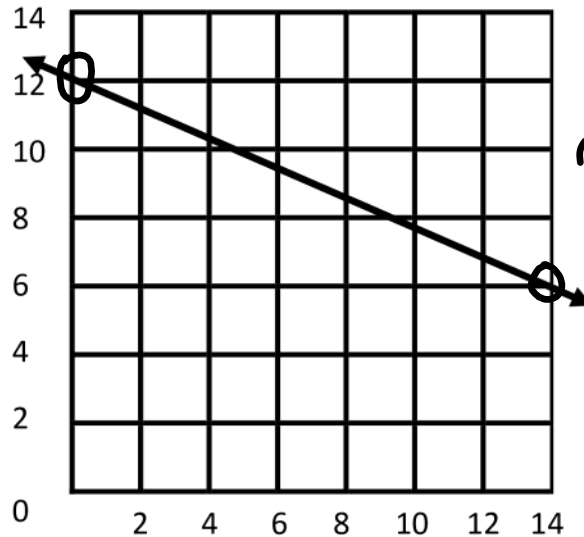
Write in point-slope form the equation of the line with a slope of 6 and passes through the point  $(3, -4)$ .

$$m = 6, (x_1, y_1) = (3, -4)$$

Point-Slope Form:  $(y - y_1) = m(x - x_1)$   
 $(y - (-4)) = 6(x - 3)$   
 $y + 4 = 6(x - 3)$

Problem 6

Write the equation of the line represented in the graph below in point-slope form.



$$(x_1, y_1) = (0, 12)$$

$$(x_2, y_2) = (14, 6)$$

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 12}{14 - 0} = \frac{-6}{14} = -\frac{3}{7}$$

Point-Slope Form

$$(y - y_1) = m(x - x_1)$$

$$(y - 12) = -\frac{3}{7}(x - 0)$$

Problem 7

Write  $y = -\frac{2}{3}x + 4$  in standard form.

$$\begin{array}{r} y = -\frac{2}{3}x + 4 \\ +\frac{2}{3}x \quad +\frac{2}{3}x \\ \hline \left(\frac{2}{3}x + y = 4\right) 3 \end{array}$$

$$2x + 3y = 12$$

Standard Form  $Ax + By = C$   
 $2x + 3y = 12$

Problem 8

Write  $(y - 4) = \frac{5}{6}(x + 2)$  in standard form.

$$(y-4) = \frac{5}{6}(x+2)$$

$$\left( y-4 = \frac{5}{6}x + \frac{10}{6} \right) \cdot 6$$

$$6y - 24 = 5x + 10$$

$$\underline{-5x} \quad \underline{+24} \quad \underline{-5x} \quad \underline{+24}$$

$$-5x + 6y = 34$$

Standard Form

$$Ax + By = C$$

$$-5x + 6y = 34$$

Problem 9

Write an equation for the line that contains  $(-3, 5)$  and is parallel to  $y = -2x + 3$ .

$$m = -2, (x_1, y_1) = (-3, 5)$$

Point-Slope Form  $(y - y_1) = m(x - x_1)$

$$y - 5 = -2(x - (-3))$$

$$y - 5 = -2(x + 3)$$

Problem 10

Find the equation of the line that contains (4, 2) and is perpendicular to

$$y = -\frac{1}{3}x + 2.$$

$$-\frac{1}{3} \quad r = -1$$
$$r = -1 \left(-\frac{3}{1}\right)$$
$$r = 3 = m$$

$$(x_1, y_1) = (4, 2)$$

Point-Slope Forms  $(y - y_1) = m(x - x_1)$

$$(y - 2) = 3(x - 4)$$

**If you complete the classwork early, please work on your unit 3 project. Bring Mr. Turner a laptop to login for you if you want to use a laptop to work on your project.**

Write in slope-intercept  
the line with a slope of 2  
and the y-intercept  $(0, 5)$

Write in Point-slope  
form the equation that  
goes through  $(5, 7)$  and  
 $(3, 8)$

Write  $y - 8 = \frac{-1}{2}(x - 3)$   
in Standard form