

LESSON OVERVIEW

Subject(s)	Mathematics, Technology
Topic or Unit of Study	Modular arithmetic
Grade/Level	Grade 9, Grade 10, Grade 11, Grade 12
Objective	Students will be able to write an integer, a , congruent to " b modulo n " or $a \equiv b \pmod{n}$.
Summary	

IMPLEMENTATION

Learning Context	Students are learning how to program in Python and are learning the different arithmetic operators (+, -, /, *, **, %). The most recent operator the students learned was exponents (**).
Teaching Strategies	Learning through games
Time Allotment	1 class periods. 50 Mins. per class.
Sample Student Products	
Author's Comments & Reflections	See the 'Notes' in the Guided Practice section.

PROCEDURE

Anticipatory Set	<i>Teacher leads a discussion about the different operators the students have learned in Python (excluding libraries).</i>
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Modeling	<p><u>Modular Arithmetic</u></p> <p><i>Teacher models modular arithmetic.</i></p>
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The Format

For a positive integer n , two integers a and b are said to be *congruent modulo n* , written as:

$a \equiv b \pmod{n}$, where a is the integer being divided, n is the modulus (what you are dividing by), and b is the remainder.

In other words: $a - b = x$ where x is divisible by n . We do not care how many times n goes into x ($x/n = \text{some number}$)! We are only concerned with the remainder, b .

Example 1:

$7 \equiv 1 \pmod{6}$ means $7 - 1 = 6$ which is divisible by 6.

Example 2:

What are the possible values for b ? It must be in the range from 0 to 5.

$37 \equiv 1 \pmod{6}$ means $37 - 1 = 36$ which is divisible by 6 ($36/6=6$).

Programming

In Python, the modular operator is %. Thus, $a \% n$ will return the remainder, b .

Example 3:

Enter into Python Shell: $15 \% 5$

Answer/Return: 0

Example 4:

Enter into Python Shell: $15 \% 4$

Answer/Return: 3

Modinoes Activity

Teacher models the setup and first two rounds of modinoes.

Guided Practice

Modinoes

This activity is designed for students to explore and practice modular arithmetic through play.

Setup:

- Teacher arranges desks into groups of four
- Seat students in groups of three and four at the desks
- Give each group a set of dominoes (or part of a set)
- Have students place all the dominoes so that the dots are face-down on one of the four desks (known as the stock or boneyard)

Instructions:

- Goal of the Game
 - The goal of modinoes is to score as many points as possible before the end of the game.
 - The game ends when a player's hand is empty (no dominoes).
- Draw Initial Hand
 - Each player draws 4 domino tiles (predetermined by the teacher, see 'Notes')
- Order of Play
 - Student who plays first has the next upcoming birthday of the group of students.
 - The turn order will rotate clockwise (to the right) of the first player for duration of the game.
 - The first player places one domino from the pile of face-down dominos, known as the stock or boneyard, and places the tile face-up in the middle of the table.
 - The first player now begins their turn.
- Player's Turn
 1. Player selects a domino in their hand which has the same value or number of dots/pips on it as one of the exposed tiles on the board. Double tiles (same number on both halves of the tile) can have one domino attached to each side (4 sides, 4 tiles) after it is connected on two sides (sandwiched), but only by vaules matching one of the values (2-2 connects to 2-d tiles, not 4-d tiles).
 2. The player places their domino face-up on the board aligning the matching values of the domino played and the existing domino.
 3. If the tile is a double tile (same number on both halves of the tile) is played, the tile is rotated perpendicular to the attaching tile to form a T-shape.
 4. If the player does not have any dominoes which match a value of an exposed domino, the player must continue to draw from the stock/boneyard until they draw a domino that does match a played tile.
 5. If there are no more tiles in the stock/boneyard, the player's turn ends and does not score (skip step 6). Players are no longer able to draw dominoes and cannot score unless they play a tile which already in their hand.
 6. Lastly, the player checks if they have scored.
- Scoring
 - At the end of each player's turn, the player adds the sum of values of the exposed (not connected) domino tile halves.
 - Single tiles which are not connected on one end, have the value of that end added to the sum (e.g., a chain of |5-3|3-4| has a sum of $5+4=9$).

- Double tiles count as the sum of both ends of the tile only if the double tile is attached on one side (makes a small T-shape). If connected on only two sides (addition sign or cross shape), it does not count as exposed (contributes no points). The tile counts as not exposed until a third or fourth tile has branched off of the double tile (takes the shape of a large T or large addition sign shape).
- If the sum is $\equiv 0 \pmod{n}$, where n is announced by the teacher, the player scores. The sum calculated earlier is added to the player's current score.
- End of the Game
 - The game ends when a student has played their last domino tile. No more players may play (no final/last round).
 - The student that played last updates their score, then all of the students compare their scores to determine the winner (person with the highest score).

Notes:

- Make sure that each set of dominoes given to students could add up to "congruent modulo n" within the first few rounds of play.
- The teacher may consider treating double tiles (e.g., 5-5 tile) as normal tiles and not causing the domino sequence to branch (placed perpendicular to the previous tile).
- The number of tiles drawn by players should leave at least 5 tiles in the stock/boneyard (to allow players to draw).

Independent Practice

Closure

Follow-Up

MATERIALS AND RESOURCES

Instructional Materials

Modinoes rules handout

Resources

- Materials and resources:
Multiple domino sets
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STANDARDS & ASSESSMENT

Standards

Assessment Plan

Assessment/Rubrics
