6-4: Solving Polynomial Equations

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LESSON OVERVIEW

Subject(s)	Mathematics
Topic or Unit of Study	Polynomials and Polynomial Functions
Grade/Level	Grade 10
Objective	1. Students will be able to factor polynomials using the sum and difference of squares identities. 2. Students will be able to factor polynomials using the quadratic equation. 3. Students will be able to factor polynomials using the perfect square trinomial identity. 4. Students will be able to factor polynomials using the sum and difference of two cubes identities.
Summary	

IMPLEMENTATION

Learning Context	
Teaching Strategies	Direct instruction, whiteboards
Time Allotment	1 class periods. 50 Mins. per class.
Sample Student Products	
Author's Comments & Reflections	

PROCEDURE

Anticipatory Set

Factoring Perfect Square Trinomials

$$a^2 + 2ab + b^2 = (a + b)^2 \quad \leftarrow makes\ the\ middle\ term\ positive$$

$$a^2 - 2ab + b^2 = (a - b)^2 \quad \leftarrow makes\ the\ middle\ term\ negative$$

Factoring a Difference of Two Squares

$$a^2 - b^2 = (a + b)(a - b) \quad \leftarrow removes\ the\ middle\ term$$

Sum and Difference of Cubes

$$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$$

$$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$$

Factor Using a Property (Textbook page 336 #13)

$$x^3 - 1000 = (x - 10)(x^2 + 10x + 100) \quad \leftarrow Difference\ of\ cubes\ property$$

$$\nwarrow a^3 \quad \nwarrow b^3 \quad a = x, b = 10 \qquad (a^3 - b^3) = (a - b)(a^2 + ab + b^2)$$

Modeling

Factoring by Using a Quadratic Pattern (textbook page 336 #21)

$$x^4 - 8x^2 + 7 = (x^2)^2 - 8(x^2) + 7 \quad \leftarrow Rewrite\ the\ expression\ to\ look\ like\ a\ quadratic\ expression$$

$$= s^2 - 8s + 7 \quad \leftarrow Replace\ all\ of\ the\ x^2\ terms\ with\ the\ letter\ s\ (the\ substitute\ term)$$

$$= (s - 7)(s - 1) \quad \leftarrow Factor\ the\ quadratic\ expression$$

$$= (x^2 - 7)(x^2 - 1) \quad \leftarrow Factor\ further$$

$$= (x^2 - 7)(x + 1)(x - 1) \quad \leftarrow Factoring\ a\ difference\ of\ two\ squares$$

The factored form of $x^4 - 8x^2 + 7 = (x^2 - 7)(x + 1)(x - 1)$.

Solving a Polynomial Equation (textbook page 336 #15)

$$x^3 - 27 = 0 \quad \leftarrow Rewrite\ expression\ as\ a\ sum\ of\ cubes$$

$$x^3 - 27 = (x)^3 + (-3)^3 = (x + (-3))(x^2 - (-3)x + (-3)^2) \quad \leftarrow Simplify$$

$$= (x - 3)(x^2 + 3x + 9) \leftarrow Factor\ further\ using\ the\ quadratic\ formula$$

$$\searrow a \quad \searrow b \quad \searrow c$$

$$\hookrightarrow x = \frac{-(3) \pm \sqrt{(3)^2 - 4(1)(9)}}{2(1)}$$

$$x = \frac{-3 \pm \sqrt{9 - 36}}{2} = \frac{-3 \pm \sqrt{-27}}{2} = \frac{-3 \pm \sqrt{-9 \cdot 3}}{2}$$

$$x = \frac{-3 \pm 3\sqrt{3}\,i}{2}$$

The roots are: 3, $\dfrac{-3}{2} + \dfrac{3\sqrt{3}\,i}{2}$, and $\dfrac{-3}{2} - \dfrac{3\sqrt{3}\,i}{2}$

Steps: rewrite cubic as sum of cubes, factor, simplify, quadratic formula

Solving a Higher-Degree Polynomial Equation (textbook page 337 #27)

$x^4 - 10x^2 + 9 = 0 \leftarrow Rewrite\ as\ a\ quadratic\ expression$

$(x^2)^2 - 10(x^2) + 9 = 0 \leftarrow Substitute\ s = x^2$

$s^2 - 10s + 9 = 0 \leftarrow Rewrite\ with\ the\ substitution$

$(s - 9)(s - 1) = 0 \leftarrow Factored$

$(x^2 - 9)(x^2 - 1) = 0 \leftarrow Replace\ the\ substitute\ with\ the\ original$

$(x + 3)(x - 3)(x + 1)(x - 1) = 0 \leftarrow Difference\ of\ squares\ property$

$x = \pm 1, \pm 3$

Guided Practice

<u>Factor Using a Property</u> (textbook page 336 #12, 14)

$x^3 + 64$

Answer: $(x + 4)(x^2 - 4x + 16)$

$125x^3 - 27$

Answer: $(5x - 3)(25x^2 + 15x + 9)$

Factoring by Using a Quadratic Pattern (textbook page 336 #23, 25)

$x^4 - 7x^2 + 12$

Answer: $(x^2 - 3)(x - 2)(x + 2)$

$x^4 - 1$

Answer: $(x - 1)(x + 1)(x^2 + 1)$

Solving a Polynomial Equation (textbook page 336 #17, 18)

$x^3 - 125 = 0$

Answer: 5, $\dfrac{-5 \pm 5\sqrt{3}\,i}{2}$

$2x^3 + 2 = 0$

Answer: -1, $\dfrac{1 \pm \sqrt{3}\,i}{2}$

Solving a Higher-Degree Polynomial Equation (textbook page 337 #29)

$$x^4 - 12x^2 - 64$$

Answer: $\pm 4, \pm 2i$

Independent Practice	**Homework**

Workbook 6-4 (pg 413) Practice Problems 2-16 (even), 26-40 (even)

Closure

Step-by-Step

Teacher goes through the general steps/ideas learned today.

Follow-Up

Warm-Up

Solve the equation: $x^4 + 4x^2 - 12 = 0$

Answer:

$$x^4 - 8x^2 + 16 = 0$$
$$(x^2)^2 - 8(x^2) + 16 = 0$$
$$s^2 - 8s + 16 = 0$$
$$(s - 4)(s - 4) = 0$$
$$(x^2 - 4)(x^2 - 4) = 0$$
$$(x + 2)(x - 2)(x + 2)(x - 2) = 0$$
$$x = \pm 2$$

MATERIALS AND RESOURCES

Instructional Materials

Resources

STANDARDS & ASSESSMENT

Standards

CA- California K-12 Academic Content Standards

Subject: Mathematics

Grade: Grades Eight Through TwelveThe standards for grades eight through twelve are organized differently from those for kindergarten through grade seven. In this section strands are not used for organizational purposes as they are in the elementary grades because the mathematics studied in grades eight through twelve falls naturally under disci-pline headings: algebra, geometry, and so forth. Many schools teach this material in traditional courses; others teach it in an integrated fashion. To allow local educational agencies and teachers flexibility in teaching the material, the stan-dards for grades eight through twelve do not mandate that a particular discipline be initiated and completed in a single grade. The core content of these subjects must be covered; students are expected to achieve the standards however these subjects are sequenced.

Standards are provided for algebra I, geometry, algebra II, trigonometry, math-ematical analysis, linear algebra, probability and statistics, Advanced Placement probability and statistics, and calculus. Many of the more advanced subjects are not taught in every middle school or high school. Moreover, schools and districts have different ways of combining the subject matter in these various disciplines. For example, many schools combine some trigonometry, mathematical analysis, and linear algebra to form a precalculus course. Some districts prefer offering trigonometry content with algebra II.

Area: Algebra IIThis discipline complements and expands the mathematical content and concepts of algebra I and geometry. Students who master algebra II will gain experience with algebraic solutions of problems in various content areas, including the solution of systems of quadratic equations, logarithmic and exponential functions, the binomial theorem, and the complex number system.

Sub-Strand 4.0: Students factor polynomials representing the difference of squares, perfect square trinomials, and the sum and difference of two cubes.

Assessment Plan

Homework is out of 20 points. The teacher will check 5 questions and grade each question (worth 4 points). The 5 questions will be from different sections of the assignment to avoid grading 5 questions that are the same problem-type (example: 5 problems asking to plot a linear equation) which may not adequately represent the student's progress towards the lesson's learning goal/standard.

Assessment/Rubrics

Rubrics:

1. Textbook Homework Rubric